ReaDiNet 2023: International conference on parabolic and stochastic models in mathematical biology

Inkyung AHN

Does movement toward better environments of one species always benefit survival over the other species?

In this study, we consider a Lotka–Volterra reaction–diffusion–advection model for two competing species under homogeneous Dirichlet boundary conditions, describing a hostile environment at the boundary. In particular, we deal with the case in which one species diffuses at a constant rate, whereas the other species has a constant rate diffusion rate with a directed movement toward a better habitat in a heterogeneous environment with a lethal boundary. We conclude that the species dispersion in the advection direction is not always beneficial, and survival may be determined by the convexity of the environment. This talk is based on joint work with Kwangjoong Kim and Wonhyung Choi.

Goro AKAGI

Rate of convergence to asymptotic profiles for fast diffusion on domains via energy methods

This talk is concerned with the Cauchy-Dirichlet problem for fast diffusion equations on bounded domains. It is well known that every (possibly sign-changing) weak solution vanishes in finite time at the unique power rate, and therefore, asymptotic profiles for such vanishing solutions are defined as a limit of rescaled solutions, which solve the Cauchy-Dirichlet problem for a fast diffusion equation with a blow-up reaction. Asymptotic profiles are characterized as nontrivial equilibria of the rescaled problem (see pioneer works of Berryman and Holland in 1980s and subsequent results for qualitative results). Recently, Bonforte and Figalli (CPAM, 2021) established an important quantitative result of the convergence of rescaled solutions to positive asymptotic profiles. More precisely, they proved an exponential convergence of nonnegative rescaled solutions to nondegenerate positive asymptotic profiles in a weighted L^2 space with a sharp rate (in view of some linearized problems) by developing a nonlinear entropy method. In this talk, we shall develop a different approach to prove exponential convergence with rates for nondegenerate asymptotic profiles with definite or changing signs. In particular, if we restrict ourselves to nondegenerate positive asymptotic profiles, we shall exhibit an H_0^1 convergence with the sharp rate. Our method of proofs is based on an energy method rather than entropic one, and a key ingredient is a quantitative gradient inequality established based on an eigenvalue problem with weights, which was already introduced by Bonforte and Figalli and also plays a crucial role in our analysis.

Sung Woong CHO

The monotone traveling wave solution of a bistable three-species competition system via unconstrained neural networks

In this talk, we present a method for approximating traveling wave solutions of the bistable three-species competition system using artificial neural networks. The given system has a unique traveling wave solution under the assumption on the solution's monotonicity. Traditional monotonic neural networks are not smooth enough to accurately represent solutions to differential equations. To address this issue, we propose a method for approximating monotonic solutions using neural networks that are constructed as primitive functions of other positive neural networks. Operator learning based neural networks in the numerical integration step can overcome the issue of differentiability with theoretical results that a small training loss implies convergence to a true solution. The experimental results show the effectiveness of our method when the wave speed is identical to zero, and that our monotonic neural network can accurately estimate both the speed and solution of the wave in these cases.

Marie DOUMIC

Asymptotic inverse problems for a depolymerising system

Shrinkage, either through depolymerisation (i.e., progressive shortening) or through fragmentation (breakage into smaller pieces) are dynamical phenomena which appear in many applications. More specifically, the departure point of our research has been protein fibrils depolymerisation, thought to be a key mechanism for many diseases (Parkinson's, Alzheimer's...) as well as many functional biomolecular systems (actin filaments). The dynamic nature of the experiments, as well as their nanoscale, makes it very challenging to estimate their features.

In this talk, we consider a model of discrete depolymerisation, originally based on a Becker-Döring model. Our aim is to estimate the initial distribution when measuring the dynamics of the first or second moment of the distribution during the depolymerization process.

We first evaluate the impact of using continuous approximations of the initial discrete model to solve this inverse problem. At first order, the model is approximated by a backward transport equation, for which the inverse problem turns to be mildly ill-posed (of order $k+1$ when used to invert the time evolution of the k-moment of the solution). This remains true when polymerisation is also considered, as in the full Becker-Döring system, though the inversion reveals more intricate due to the fact that the problem becomes nonlinear.

At second order, the asymptotic model becomes an advection-diffusion equation, where the diffusion is a corrective term, complemented with an original transparent boundary condition at $x = 0$. This approximation is much more accurate, but we face a classical accuracy versus stability trade-off: the inverse reconstruction reveals to be severely ill-posed. Thanks to Carleman inequalities and to log-convexity estimates, we prove observability results and error estimates for a Tikhonov regularisation. We then develop a Kalman-based observer approach, which reveals very efficient for the numerical solution.

This is a joint work with Philippe Moireau (Inria), inspired by depolymerisation experiments carried out by Human Rezaei and collaborators (Inrae).

Arnaud DUCROT

Threshold Effect for a bistable equation with nonlocal diffusion

In this talk we are interested in the threshold phenomena for propagation in nonlocal diffusion equations with some compactly supported initial data and bistable nonlinearity. We provide quantitative estimates for such phenomena. The outcomes dramatically depend on the tails of the dispersal kernel and can take a large variety of different forms. The strategy is to combine sharp estimates of the tails of the nonlocal diffusion kernel and the construction of accurate sub- and super-solutions.

Leo GIRARDIN

Non-local pulling in reaction-diffusion equations

In recent years it has been established that, in reaction–diffusion models of monostable type, a favorable environment that flees away from a population density arising from a localized initial condition can in fact improve the spreading speed of this population density. In a simplified Fisher–KPP setting, when the speed of the favorable area is sufficiently small for the population to keep up, then the spreading speed is correctly predicted by the Fisher–KPP speed in this environment. But when the speed is too large for the population to keep up, the Fisher–KPP speed of the second, less favorable environment, is in general only a lower estimate for the spreading speed: the population can be "non-locally pulled" by its exponential tail in the favorable area, even though the distance between this area and the spreading front is linearly increasing in time. The exact spreading speed is given by an explicit formula and at least two methods of proof are known. In this talk I will review some results about non-local pulling in reaction–diffusion systems and equations and then I will present a work in progress in collaboration with Thomas Giletti and Hiroshi Matano.

Ludovic GOUDENEGE

Numerical approximation of SDEs with distributional drift

We consider a stochastic differential equation arising in mathematical biology, especially an ordinary differential equation perturbed by additive noise. It is natural to add a reflection on the boundary of the physical domain to avoid unexpected behavior like negative concentration. It is also natural to create different dynamics in different spatial domains with a non-smooth boundary between them by adding indicator functions in the drift of the ODE. However, such irregular terms could destroy essential mathematical properties like the existence or uniqueness of the solution.

We will present in this talk how we can define a unique solution to these very singular SDEs, allowing us to consider indicator functions but also singular potential like $x^{-\alpha}$, Dirac forces on points or surfaces, etc. The theoretical result is obtained by a tamed Euler-Maruyama scheme, which also provides a numerical technic to simulate the solution and convergence speed. These results will be associated with numerical simulations illustrating the algorithm's efficiency and the optimality of convergence speed.

Jong-Shenq GUO

Stability of traveling waves in reaction-diffusion systems with equal diffusivities

In this talk, we first present a Liouville type theorem for entire solutions of a class of reaction-diffusion systems with equal diffusivities. Then, based on this Liouville type theorem, we derive a stability theorem for traveling waves of these reaction-diffusion systems. Finally, we apply this stability theorem to some predator-prey models. This talk is based on a recent joint work with Masahiko Shimojo from Tokyo.

Benoit HENRY (canceled)

Time-reversal of the spine of branching processes near stationarity

It is usual to approximate PDE in biology using branching processes as microscopic models. The behavior of such processes, when linear, can then be investigated using tools such as many-to-one formulas and the associated spine processes. This process is typically a time-inhomogeneous Markov process. In this talk, we will discuss some homogenization phenomenon arising when approximating a PDE at stationarity for the time-reversed spine process.

Chun-Hsiung HSIA

Synchronization of Kuramoto oscillators with the distributed time-delays and inertia effect

This is joint work with Chang-Yeol Jung, Bongsuk Kwon and Sunghwan Moon. In this talk, we prove the complete and partial phase synchronization for the Kuramoto oscillators with distributed time-delays and inertia effect. Our results assert that the Kuramoto models incorporated with a small variation of distributed time-delays and inertia effect still exhibit synchronization. This shows the robustness of the synchronization phenomena of the original Kuramoto model in the perturbation of time-delay and inertia effects.

Chih-Chiang HUANG

Traveling Waves of a Gradient System with Three-Wells

In the talk, I would like to study traveling waves of a gradient system in an infinite cylindrical domain. On the bounded cross section of the cylinder, we assume that the system exactly has three local minimizers with 3 different or 3 equal energy level. These two cases will be discussed in this talk. Applying a variational approach, the existence of traveling waves connecting two local minimizers will be constructed. Moreover, the concept of weakly wave interaction will be also involved to study two waves with the same speed.

Jinsu KIM

Inferring the underlying network for a small size biochemical system in a well-mixed enviroment using the intrinsic noise and discrete nature

When an underlying reaction network is given for a biochemical system, the system dynamics can be modeled with various mathematical frameworks such as continuous-time Markov processes. In this manuscript, the identifiability of the underlying network structure with a given stochastic system dynamics is studied. It is shown that some data types related to the associated stochastic dynamics can uniquely identify the underlying network structure as well as the system parameters. The accuracy of the presented network inference is investigated when given dynamical data is obtained via stochastic simulations.

Yong-Jung KIM

Fractionation by diffusion; models and experiments

Various fractionation phenomena by diffusion, e.g., the Soret effects, contradict the common belief that random particle movements eventually homogenize the concentration. We show that fractionation is the nature of a random movement, but not because of an external force. Experimentally, the fractionation process is measured across a solid-solid interface. By comparing experimental data and the simulation of diffusion laws, we conclude that the diffusivity alone cannot explain fractionation by diffusion, but we need two parameters of motility and conductivity.

Min-Gi LEE

A limit of microscopic dweller-wanderer system to a chemotaxis model

In this talk, we study a derivation of a chemotaxis model where species exhibits adaptive diffusion depending on the local information of population and food density. More specifically, the flux of population is the negative gradient of $\gamma(m)u$, where m is the food density, u is the population density, and $\gamma(\cdot)$ is a decreasing function. This type of flux law has attracted considerable attention in the reaction-diffusion community. Funaki, Mimura, and Urabe (2012) presented a derivation by hydrodynamic limit of a kinetic model where species may exists either as a fast diffusive or a slow diffusive mode; they convert to each other quickly to meet the the quasi-equilibrium ratio between them. One small concern is that there has been an employment of asymptotic power laws $\gamma(m) = \frac{1}{1+m^p}$ in literature while the derived $\gamma(m)$ is bounded from below by the diffusion constant of slow diffusive mode, away from 0. To fit the power laws in, one must drop the diffusion term of the slow mode, which might cause technical challenges. We show that the limit presented by Funaki, Mimura, and Urabe is still done the same with 0 diffusive dwelling mode. (This is joint work with Kyunghan Choi.)

Pierre MAGAL

A Holling's predator-prey model with handling and searching predators

The goal of this presentation is to explain how to derive the classical Rosenzweig-MacArthur's model by using a model with two groups of predators in which we can separate the vital dynamic and consumption of prey to describe the behavior of the predators. This will be especially very convenient if we want to add an age or size structure to the predator population. As mentioned by Holling (without mathematical model), we divide the population of predators into the searching and the handling predators. In this article we study some properties of this model and conclude the paper proving that the model con- verges to the classical Rosenzweig-MacArthur's model by using an appropriate rescalling. This convergence property is observed by using numerical simulations. We also apply this model to the Canadian snowshoe Hares and the Lynx.

Reference: S.-B. Hsu, Z. Liu, and P. Magal (2020) A Holling's predator-prey model with handling and searching predators, SIAM Journal on Applied Mathematics, Vol. 80, No. 4, pp. 1778-1795.

Yoshihisa MORITA

Front propagation for the bistable reaction-diffusion equation on unbounded metric graphs

We are concerned with a bistable reaction-diffusion equation on unbounded metric graphs. Specifically, we study entire solutions exhibiting the front propagation on tree-like graphs which consists of several half-lines. We give a condition for the propagation beyond the branching points in a specific symmetric tree graph. The main result is given by a recent joint work with Shuichi Jimbo (Hokkaido University).

Ken-Ichi NAKAMURA

Front propagation and blocking for the Lotka-Volterra strong competition system in an infinite star graph

We consider the 2-component Lotka-Volterra competition-diffusion system in an infinite star graph with a single junction. Under strong competition conditions, we give sufficient conditions for the success/failure of the invasion of superior species beyond the junction. The method is based on a standard argument by constructing super-subsolutions with the help of a new result on the speed of traveling waves for the Lotka-Volterra competition-diffusion system on the whole line. This talk is based on joint work with Yoshihisa Morita (Ryukoku University) and Toshiko Ogiwara (Josai University).

Kei NISHI

Pulse bifurcations in a three-component FitzHugh-Nagumo system

Pulse dynamics in a three-component FitzHugh-Nagumo system in one dimensional space is considered. The system admits a pulse solution of bistable type, which exhibits a variety of interface dynamics, not observed for the two-component FitzHugh-Nagumo system. In order to analytically investigate the mechanism for the pulse behavior, we apply the multiple scales method to the original reaction-diffusion system, and derive finite-dimensional ordinary differential equations which describe the motions of the pulse interfaces. The reduced ODEs enable us to reveal the global bifurcation structures of the pulse solutions, and to clarify the mechanism behind the variety of the pulse dynamics from a view point of bifurcation theory.

Toshiyuki OGAWA

Alien invasion into the buffer zone between two competing species

Bifurcation of non-monotone traveling wave solutions of the three-species Lotka-Volterra competition diffusion system under strong competition is studied. The well-known front and back traveling wave formed by two species may lose its stability by the effect of third species and, as a result, allows the invasion. To discuss how the invasion is possible, stability change with respect to the intrinsic growth rate for the alien species are studied. Both numerical and theoretical bifurcation analysis around the bifurcation point reveal how the invasion affects the segregation of the original two species. We also discuss global bifurcation structure of the invading travelling wave solution focusing on the problem how the small invasion becomes large and dominant.

Lionel ROQUES

2 be 3 or not to be?

We propose a model to describe the adaptation of a phenotypically structured population in a H-patch environment connected by migration, with each patch associated with a different phenotypic optimum. We show that persistence or extinction depends on the sign of a principal eigenvalue, λ_H, and we study the effect of increasing the number of patches on the persistence of a population. In particular, we propose a precise characterisation of the situations where the addition of a third patch increases or decreases the chances of persistence, compared to a situation with only two patches. This study has implications in agroecology and for understanding zoonoses, where intermediate hosts are commonly implicated in outbreaks in human populations (e.g., for influenza A or coronaviruses). From a mathematical point of view, it is based on a fixed point theorem, comparison principles, integral estimates, variational arguments, rearrangement techniques, and numerical simulations.

Akira SASAKI

Mathematical Models Describing Evolutionary Immune Escapes of Pathogens

Despite the propensity for complex and non-equilibrium dynamics in nature, eco-evolutionary analytical theory typically assumes that populations are at equilibria. In particular, pathogens often show antigenic escape from host immune defenses, leading to repeated epidemics, fluctuating selection, and diversification. We model the evolutionary chase and escape of pathogen antigenicity and host immune system by using a reaction-diffusion system in antigenicity space. The system describes the pathogen immune escape as coupled traveling waves of pathogen antigenicity and host specific-immunity. Our analysis predicts how the speed of antigenic escape of pathogen (traveling wave speed) depends on epidemiological and genetic parameters, as well as the condition under which a stable traveling wave becomes destabilized, resulting in periodic bursts of pathogen outbreaks both in time and antigenicity space. Our model also predicts how this antigenic escape impacts the evolution of transmission and virulence of a pathogen. An extended model incorporating the effect of heterogeneity in host immune competence is also analyzed, showing that the presence of immunocompetent hosts sensitively speeds up the evolution rate of antigenic escape.

Philippe SOUPLET

Blow-up profiles for the parabolic-elliptic Keller-Segel system in dimensions $n \geq 3$ and related problems

We study the blow-up asymptotics of radially decreasing solutions of the parabolic-elliptic Keller-Segel system in space dimensions $n \geq 3$. In view of the biological background of this system and of its mass conservation property, blowup is usually interpreted as a phenomenon of concentration or aggregation of the bacterial population. Understanding the asymptotic behavior of solutions at the blowup time is thus meaningful for the interpretation of the model. Under mild assumptions on the initial data, for $n \geq 3$, we show (in collaboration with Michael Winkler) that the final profile of the singularity behaves like $|x|^{-2}$, and we also obtain refined space-time estimates. This is in sharp contrast with the two-dimensional case, where solutions are known to concentrate to a Dirac mass at the origin. I will also present results on the asymptotic blow-up behavior for other nonlinear parabolic problems that can be obtained by related methods.

Tsubasa SUKEKAWA

Linearized eigenvalue problems for a mass-conserved reaction-diffusion compartment model

In a mass-conserved reaction-diffusion system, we can observe by numerical simulations that a transient pattern such as a stripe one converges to a spatially monotone profile, for example, a unimodal pattern. To understand the dynamics theoretically, we introduce a reaction-diffusion compartment model. The model equation is defined on multiple regions (compartments), and each compartment is connected by diffusive coupling based on Fick's law. In this talk, we analyze linearized eigenvalue problems of spatially non-monotone stationary solutions in the mass-conserved reaction-diffusion compartment model, and construct eigenvalues and eigenfunctions by the implicit function theorem. We also show that this allows us to theoretically understand the mechanism for the instability of non-monotone stationary solutions. (This talk is based on joint research with Prof. Shin-Ichiro Ei in Hokkaido University)

Chi-Jen WANG

Families of critical droplet solution for non-equilibrium phase transitions in lattice bistable systems

Discontinuous phase transitions are common in the steady states of diverse non-equilibrium systems describing catalytic reaction-diffusion processes, biological transport, spatial epidemics, etc. These transitions are usually associated with equistability of two stable states, as can be determined by stationarity of a planar interface separating these states. For equilibrium systems, this criterion is equivalent to the Maxwell construction determining coexistence of two states at a unique equistability point. Analyses of nucleation phenomena near such transitions aims in part to characterize critical droplets of the more stable state embedded in the less stable metastable state, where these droplets correspond to stationary curved interfaces between the two states. There is a range of critical droplets and their critical sizes are expected to diverge when approaching the transition. The analysis of discontinuous transitions in spatially discrete non-equilibrium systems also reveals an interface propagation failure. We show that this feature, together an orientation dependence of planar interface propagation also deriving from spatial discreteness, results in the occurrence of entire families of stationary droplets. The extent of these families increases approaching the transition and can be infinite if propagation failure is realized. In addition, there can exist a regime of generic two-phase coexistence where arbitrarily large droplets of either phase always shrink. Such rich behavior is qualitatively distinct from that for classic nucleation in equilibrium and spatially continuous non-equilibrium systems.

Feng-Bin WANG

The basic reproduction ratio for an ecological model with internal nutrient storage

In this talk, we first introduce a variable-internal-stores model in a flowing habitat with temporal-spatial variations, where two essential nutrients are stored internally within individuals, and population growth is a positive function of stored nutrients. The main difficulties in mathematical analysis are caused by the minimum of two Droop functions and the singularity at the extinction state. The threshold type result on the extinction/persistence of the species can be determined by the basic reproduction ratio $\mathcal{R}_0$, which is defined by a time-periodic homogeneous evolution system. We will also investigate the principal eigenvalue of the associated homogeneous eigenvalue problem, and show that the sign of $\mathcal{R}_0 - 1$ determines the stability of the extinction state of our system. This talk is based on a joint work with Drs. Lei Zhang and Xiao-Qiang Zhao.

Chang-Hong WU

Classification of the spreading behaviors of a two-species diffusion-competition system with free boundaries

In this talk, we revisit the spreading behavior of two invasive species modeled by a diffusion-competition system with two free boundaries in a radially symmetric setting, where the reaction terms depict a weak-strong competition scenario. Numerical simulations suggested that for all possible initial states, only four different types of long-time dynamical behaviors can be observed: (1) chase-and-run coexistence, (2) vanishing of u with v spreading successfully, (3) vanishing of v with u spreading successfully, and (4) vanishing of both species. Here we rigorously prove that, as the initial states vary, there are exactly 0̆00Cfive types of long-time dynamical behaviors: apart from the four mentioned above, there exists a 0̆00Cfifth case, where both species spread successfully and their spreading fronts are kept within a finite distance to each other all the time. This talk is joint work with Professor Yihong Du.

Changwook YOON

Bacterial traveling wave phenomenon induced by exact cross-diffusion and population growth

In this talk, under the assumption of describing realistic bacteria behavior, we consider a chemotaxis system with cross-diffusion and population growth. By a singular limit procedure, we prove that a linear system with active and inactive cells approximates the original system. Moreover, it turns out that population dynamics can only produce a monotone traveling wave, and thus the chemotactic dynamics is necessary for bacterial traveling band phenomena. We will investigate the developed models by comparing numerical simulations and experimental data.